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TECHNICAL REPORT

SOME QUANTUM CONSIDERATIONS FOR SUBMILLIMETER AND OPTICAL COMMUNICATIONS

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TECHNICAL REPORT

SOME QUANTUM CONSIDERATIONS FOR SUBMILLIMETER AND OPTICAL COMMUNICATIONS

by

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WORK CARRIED OUT AS PART OF THE LOCKHEED INDEPENDENT RESEARCH PROGRAM

Lockheed

MISSILES & SPACE COMPANY

A GROUP DIVISION OF LOCKHEED AIRCRAFT CORPORATION

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FOREWORD

Lockheed Missiles & Space Company sponsors a program of independent investigation directed toward more effective use of communication channels and techniques. The material in this report, which was first presented at the IRE-PGMTT Millimeter and Submillimeter Conference, 7 – 10 January 1962, in Orlando, Florida, is concerned with communication techniques for submillimeter- and optical-wavelength channels.

ABSTRACT

Error probabilities and bounds on information transmission capacities are determined for some communications techniques applicable to channels with significant quantum disturbances. Consideration is given to the performance of systems using ideal energy detection with and without signal preamplification. The consequences of using signals of limited coherence are evaluated in several cases. It is found that quantum disturbances do not inherently limit the information that can be communicated over a channel in the manner that additive noise does.

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ILLUSTRATIONS

Section 1
INTRODUCTION

1.1 SIGNIFICANCE OF QUANTUM EFFECTS

The effects of energy quantization in electromagnetic fields have become increasingly important as attention is shifted to communication potentials of submillimeter and optical wavelengths. Recent articles by Stern (Refs. 1, 2) and Gordon (Refs. 3, 4) provide a foundation for treating information-transmission capacities and digital-system error probabilities in terms of the allowed discrete states of communications signals. In this paper the channel model proposed by Gordon (Ref. 3) is used for analysis of some attractive elementary communication techniques and for comparing their performance with that of theoretically optimum systems.

At wavelengths of a few microns or less, quantization phenomena may be a greater source of uncertainty in a communication channel than additive thermal noise. At such wavelengths it is found that the proposed digital modulation techniques are not necessarily degraded by gross imperfections in the degree of coherence of transmitting signal sources. Furthermore, where quantum effects are the primary source of channel uncertainty, these proposed techniques are found to be potentially superior to any system of modulation which requires, or uses, linear amplification prior to received-signal detection.

1.2 PHOTON MODEL OF A COMMUNICATION CHANNEL

Gordon has suggested that a communication signal may be described by a set of integers, each of which indicates the state of excitation (in integral numbers of photons, each of energy hf) of a particular degree-of-freedom (dof) of that signal. Each dof has

time-frequency dimensions of roughly Δt by Δf, where $\Delta t = 1/\Delta f$, in the particular signal expansion described by Gordon (Ref. 3). Such a signal representation is consistent with the Planck relation $E = hf$, with the Heisenberg uncertainty principle, and with the nonindependence of measured electric and magnetic fields that arises in quantum mechanics.

Much of Shannon's mathematical theory of communication (Ref. 5) for discrete channels may be applied to a channel model reflecting integral excitation numbers per dof of the transmitted and received signals. In applying the theory, however, one must appreciate that certain special properties of energy-based signal descriptions do not necessarily obey the combinational laws of the more familiar amplitude descriptions. For example, the linear superposition of two (amplitude) electromagnetic fields is not characterized correctly by the algebraic sum of their corresponding dof excitation numbers. Thus "additive" noise cannot be treated as an additive signal perturbation within the framework of the proposed photon-channel model.

In the discussions which follow, it is assumed that the percentage bandwidths of signals are small so that photon energies and thermal-noise power spectral densities may be regarded as uniform within the frequency ranges involved.

1.3 TYPES OF SIGNAL SOURCES

Message information may be encoded for transmission over a channel by a prearranged transformation, or mapping, of message content into a set of dof excitation numbers. Where a transmitted signal source has a high degree of "coherence", or predictability, it may be possible to generate a signal conforming with considerable accuracy to the desired excitations. However, since jitter, frequency instability, time uncertainty, and fluctuations of carrier-signal power and associated noise, etc., tend to limit the achieved degree of coherence, the precision with which a specified excitation may be imparted to a specified dof may be significantly limited in many

practical cases. Some promising signal generators* might even be classed as sources of band-limited noise, which exhibit large random fluctuations of dof excitation numbers about controllable mean values. Where such excitation uncertainties exist in available signal sources, it may be desirable to encode message information in terms of average excitations of reasonably large numbers of adjacent degrees-of-freedom, rather than in terms of individual dof excitations.

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* A very wideband example is the presently available gallium arsenide light-emitting diode.

Section 2
DESCRIPTION OF THE PROPOSED SYSTEMS

Primary consideration is given here to M-ary digital systems involving frequency-shift and/or pulse-position-shift modulation. A pulse of carrier is transmitted in one of M available non-overlapping (in frequency and time) positions, or sectors, of time-frequency space. The different possible sectors are assumed to have equal a priori probabilities so that correct receiver determination of pulse location conveys $\log_2(M)$ bits of information.

Two types of receivers are considered. The first uses passive frequency filters followed directly by time-gated energy detectors to observe the total received-signal energy (in integral numbers of photon-energy units) of each possible received-signal sector. Special noiseless quantum-counting devices, such as described by Siegman (Ref. 6) and others, are particularly desirable for this kind of receiver. The second receiver employs linear amplification of the received signals followed by a detection scheme similar to that of the first receiver, except that, as a result of the preamplification, active filters, heterodyne techniques, and a variety of detection methods can be employed with little regard to the noise which such devices introduce.

It may be asked why one doesn't merely amplitude modulate and detect signals at optical frequencies using relatively conventional modulators and envelope detectors. Clearly this can be done, but the efficiency of the system, as expressed in bits of information per unit of received energy, may be vastly poorer than that of the techniques considered in this paper. If efficiency is of no importance, then techniques obviously may be chosen primarily on the basis of convenience and simplicity for particular applications.

To illustrate just how inefficient a communication system might be, consider the example analyzed by Gabor (Ref. 7) a number of years ago. For a system employing both amplitude and phase modulation, and using a detection scheme based on interaction of a received signal with an electron beam, Gabor showed that a single determination of amplitude and phase to approximately one percent accuracy each required about 10^8 received photons, even in the total absense of additive noise. This result corresponds to nearly 10^7 photons per bit of received information, compared to a fraction of a photon per bit of received information that is theoretically possible with some of the techniques considered in this paper. Gabor's example made very efficient use of bandwidth, achieving nearly 13 bits/sec/cps, but the resulting 7 orders of magnitude degradation in power economy is perhaps more significant at a wavelength so generously endowed with bandwidth.

Section 3
EVALUATION OF ERROR PROBABILITIES

3.1 USE OF A NOISE-LIKE SIGNAL SOURCE

To illustrate the consequences of limited signal-generator coherence, consideration is first given to the use of a thermal-noise signaling pulse of bandwidth B having mean power S_T and having duration $T = q/B$, where q is an integral number of degrees-of-freedom pertaining to each available pulse-location sector. As shown by Stern (Ref. 1) and Gordon (Ref. 3), the dof excitation numbers of such a signal are random and independent and are described by the discrete exponential probability distribution

$$P(n_T)\Big|_{\bar{n}_T} = \left(\frac{1}{\bar{n}_T + 1}\right)\left(\frac{\bar{n}_T}{\bar{n}_T + 1}\right)^{n_T} \tag{3.1}$$

where $\bar{n}_T$, equal to $S_T \div hfB$, is given as the mean excitation per dof, which can be regulated at the transmitter. After mean attenuation A and the addition of random thermal noise of average excitation $\bar{n}_N$ per dof, the signal is available for observation at the receiving station, as indicated in Fig. 1.

It has been shown by Shimoda, Takahashi and Townes (Ref. 8) that an exponential distribution of photons such as Eq. (3.1) retains its exponential character with a new mean value of $\bar{n}_I = A\bar{n}_T$ after experiencing attenuation. Superposition of the additive

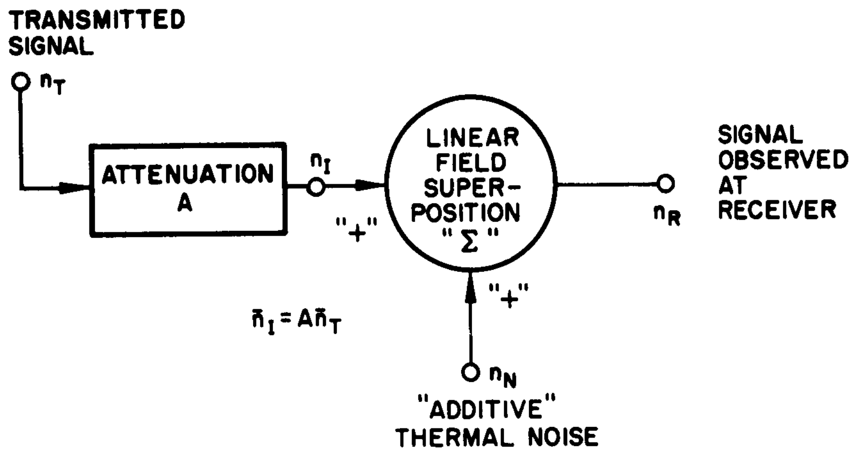

Fig. 1 Communication Channel Model.

thermal noise n_N results in received signal excitation n_R, which must also have the statistics of thermal noise. Thus,

$$P(n_R)\Big|_{\bar{n}_T,\bar{n}_N} = \left(\frac{1}{A\bar{n}_T + \bar{n}_N + 1}\right)\left(\frac{A\bar{n}_T + \bar{n}_N}{A\bar{n}_T + \bar{n}_N + 1}\right)^{n_R} \tag{3.2}$$

It should be noted that Eq. (3.2) is also applicable to the excitation numbers of pulse-location sectors where no pulse is transmitted, except that here $\bar{n}_T$ is equal to zero rather than equal to S_T/hfB. In Eqs. (3.1) and (3.2) we have merely used energy-state descriptions of random signals rather than the familiar continuous gaussian-amplitude-approximating descriptions that are not entirely adequate for the present purposes.

3.1.1 The Quantum-Counting Detector.

An optimum probability computing receiver for the described configuration may be interpreted as being one which measures and totals the excitations of the q degrees-of-freedom associated with each of the M sectors available to a given pulse. Probability of decision error is minimized by "guessing" that the pulse lies in that sector having the greatest total energy. Since the energy measurements must be in discrete multiples of photon energy, equal measurements in two or more sectors are possible, and the resulting ambiguity must be resolved in some way – a random selection of one of the sectors having greatest energy is assumed herein.

In calculating error probabilities for the systems described, one may first determine the probability distribution for the total number of photons observed in a received-pulse sector when the pulse is present and when it is absent. Since the different dof excitations are statistically independent, as previously noted, the probability distribution $P_q(n_R)\big|_{\bar{n}_T,\bar{n}_N}$ of the sum of the excitations of q degrees-of-freedom of a particular sector may be obtained by a discrete convolution of $P(n_R)\big|_{\bar{n}_T,\bar{n}_N}$ with itself a total of $(q-1)$ times.*

The probability expressions for receiver decision error are complicated somewhat by the large number of possible ways that an error can arise. Decision-error probability corresponds to the probability that a random sample taken from the distribution $P_q(n_R)\big|_{\bar{n}_T,\bar{n}_N}$ will be rejected in favor of a false signal i.e., one of M-1 independent random samples taken from the probability distribution $P_q(n_R)\big|_{0,\bar{n}_N}$. A correct decision will occur whenever the first sample exceeds all the others and one-half the time that the first sample is equal to one of the other samples and exceeds the remaining M-2 samples, etc. Expressed as a summation, the result assumes the form

*For computational purposes it may be convenient to perform this multiple convolution by Z-transformation followed by exponentiation.

$$\epsilon = 1 - \underbrace{\sum_{n_R=0}^{\infty} P_q(n_R)\Big|_{\bar{n}_T,\bar{n}_N}}_{\substack{\text{Probability that}\\ \text{occupied sector}\\ \text{has } n_R \text{ photons}}} \sum_{i=0}^{M-1} \frac{1}{i+1} \underbrace{\left[P_q(n_R)\Big|_{0,\bar{n}_N}\right]^i}_{\substack{\text{Probability}\\ \text{that } i \text{ vacant}\\ \text{sectors also}\\ \text{have } n_R\\ \text{photons}}} \underbrace{\left[\sum_{\gamma=0}^{n_R-1} P_q(\gamma)\Big|_{0,\bar{n}_N}\right]^{M-1-i}}_{\substack{\text{Probability that the}\\ \text{remaining } M-1-i\\ \text{vacant sectors have}\\ \text{less than } n_R\\ \text{photons}}} \tag{3.3}$$

where: $\sum_{\gamma=0}^{-1} P_q(\gamma) \triangleq 0$.

In most practical situations there is negligible probability of having more than two equal, maximum energy measurements so that terms of the above summation for $i > 2$ may be neglected.

At extremely high frequencies and moderate or low background noise temperatures, the average energy per d o f of additive noise sources may become vary small relative to photon energy hf so that $\bar{n}_N \ll 1$ and $P(n_N = 0)$ approaches unity, as shown by the form of the exponential probability distribution of Eq. (3.1)*. In such a situation, which is henceforth designated "noiseless", Eq. (3.3) degenerates to a particularly simple form,

$$\epsilon = \left(\frac{M-1}{M}\right)\left[P_q(0)\Big|_{\bar{n}_T,0}\right] \tag{3.4}$$

*For example, at 6×10^{14} cps and 300° K, thermal radiation is equivalent to an average of one photon for every 10^{40} degrees of freedom.

which, for the exponentially distributed source of Eq. (3.2), yields

$$\epsilon = \left(\frac{M - 1}{M}\right)\left(\frac{1}{A\bar{n}_T + 1}\right)^q \tag{3.5}$$

where q is again the number of d o f comprising a pulse sector.

Some results obtained from Eqs. (3.3) and (3.5) are presented in a subsequent section.

3.1.2 Energy Detection After Preamplification.

An interesting alternate, but suboptimum, detection technique for the described system involves low-noise preamplification of received signals prior to energy measurements. Such preamplification appears to be mandatory for certain types of detectors and for some kinds of modulation. It may be instructive to determine how much degradation occurs when the best possible receiving preamplifier is used in connection with the system that has been described.

Shimoda, Takahasi, and Townes (Ref. 8) have shown that the output of an ideal linear physical amplifier (having inherent spontaneous photon emission and negligible photon absorption) is characterized by the exponential probability distribution

$$P(n_A)\Big|_{\bar{n}_T, \bar{n}_N} = \left(\frac{1}{\bar{n}_A + 1}\right)\left(\frac{\bar{n}_A}{\bar{n}_A + 1}\right)^{n_A} \tag{3.6}$$

with $\bar{n}_A = G(\bar{n}_R + 1) - 1 = G(A\bar{n}_T + \bar{n}_N + 1) - 1$

and G = mean power gain of amplifier, when the distribution of input photons is exponential, as given in Eq. (3.2). Provided G is large, the output signals of such

an amplifier are accurately described by the familiar continuous gaussian noise representation. The signal observed in the pulse-occupied sector after amplification thus corresponds to a classically amplified sum of a classical thermal-noise pulse of man power $AP_T = A\bar{n}_T hfB$ and equivalent additive classical noise of total mean power $(\bar{n}_N + 1 - 1/G)\, hfB \approx (\bar{n}_N + 1)\, hfB$. $\bar{n}_N$ may now be identified as the discrete equivalent to the power-spectral density of the additive classical noise field, expressed in units of photons/sec/cps. Similarly, hf may be identified as the effective additive-noise power-spectral density introduced by quantum effects, including spontaneous emission. The foregoing results offer detailed corroboration, in the present instance, of Gordon's conclusion (Ref. 3) that the simple addition (voltagewise) of the amplified effective input quantum noise (of power hfB) to a classically amplified signal and input noise accounts for all fluctuations in the output wave of an ideal physical amplifier.

An analysis of error probabilities for the proposed system, when it employs noise-like transmitter signals and receiver preamplification, may be taken directly from results applicable to certain classes of badly-behaved scatter-communication channels (Ref. 9) which produce noise-like received signals no matter what kind of signals are transmitted. For this purpose it is only necessary to introduce a total effective noise power-spectral density given by

$$n'_o = n_o + hf = \bar{n}_N + hf \qquad , \quad (3.7)$$

so that

$$\mathrm{SNR}_{\mathrm{effective}} = \frac{AP_T}{(n_o + hf)B} \quad . \qquad (3.8)$$

Some results of such a procedure are included in a later section.

3.2 USE OF A COHERENT SIGNAL SOURCE

For an ideally-coherent carrier signal source the bandwidth after pulse modulation will be approximately 1/T, and each pulse may be characterized by the excitation of a single dof. Since the number of photons in a coherent transmitted pulse may be known accurately (though not exactly), the probability distribution of photons received after attenuation is given by the Poisson distribution (Ref. 3)

$$P(n_I)\Big|_{\bar{n}_T} = \frac{(A\bar{n}_T)^{n_I}}{(n_I)!} e^{-A\bar{n}_T} \tag{3.9}$$

Where the receiver uses ideal physical preamplification, the received signals and noise are again accurately represented by their classical equivalents, provided that an extra additive gaussian-noise term of power hfB is introduced. With a simple change of scale in received signal power (or energy), by the factor $(n_o + hf)/n_o$, published performance curves for a wide variety of particular modulation and detection techniques become immediately applicable to channels having significant quantum disturbances.

Where photon-counting detectors are used without prior signal amplification, the analysis is not always handled so easily. An important class of cases involving negligible additive noise can be readily handled, since substitution of Eq. (3.9) into Eq. (3.4) for q equal to one degree of freedom readily yields

$$\epsilon = \left(\frac{M-1}{M}\right) e^{-A\bar{n}_T} \tag{3.10}$$

for the decision error probability of the proposed digital communication techniques. If additive noise is present such that $n_o \gg hf$, the degrading effects of ideal physical preamplification are minor; therefore, results obtained by the method of the previous

paragraph accurately reflect system performance even when detection is used without prior signal amplification. Thus, techniques are available to allow the calculation of error probabilities and related performance criteria when additive noise is either negligible or much more significant than the quantum noise disturbances.

General probability manipulations for properly combining the dof excitation probabilities of coherent signal and additive random noise have not yet been worked out. For this reason an accurate evaluation of error probabilities has not been made for the proposed system in the ideally-coherent-signal case where $n_0 \approx hf$ and photon-counting detectors are used without prior received-signal amplification. Useful performance bounds can nevertheless be obtained for such a case; such bounds are presented along with other results in a following section.

3.3 NOTE ON THE COHERENCE OF SOURCES

Additional comment may be needed concerning the narrow-band noise source considered as a possible signal carrier in this paper, and characterized by the quantum probability distribution of Eq. (3.1). Such a signal clearly may have spatial coherence across the surface of its source radiator, and yet be relatively lacking in time coherence. From a distant point, for a given received signal power, such a signal cannot be distinguished from a spatially-incoherent source of the same bandwidth and physical dimensions. The latter source would, of course, radiate in a broader beam and thus radiate greater total power, but performance criteria that are based on received energy levels would be unaffected by this difference.

Any practical signal generator has limited coherence because of random perturbations of frequency and phase. If the amplitude (or envelope) of such a source also exhibits pronounced random fluctuations, then its photon distribution will be akin to the exponential distribution of thermal noise, as given in Eq. (3.1). On the other hand, if the amplitude of such a signal is relatively constant, then the probability distribution of photons per degree-of-freedom approaches the Poisson distribution of a truly coherent signal source, as given in Eq. (3.9).

It appears at present that the individual output lines, or modes, of some continuous laser sources are of relatively contant amplitude, and hence are reasonably well approximated by Eq. (3.9). However, there usually arises a mixture of several such lines, which appear as summed outputs of independent oscillators operating on slightly different frequencies. Unless one is able to isolate these lines for individual modulation and detection, the resulting signal statistics will be much like those of random noise, as given in Eq. (3.1). Although the (relatively narrow) autocorrelation function of the composite signal tends to repeat itself at intervals, it is difficult to take advantage of such repetition. One may be forced to modulate and detect in the same manner and with essentially the same results as though the source were a band of random noise many times wider than the individual line width.

3.4 RESULTS OF PERFORMANCE ANALYSES

In the following illustrations, received-pulse energies are normalized in units of the total effective noise power-spectral density $n'_o \triangleq \bar{n}_N + hf$. Energies are divided by $\log_2(M)$ to provide normalization with respect to the information encoded in a symbol, or pulse.

Some error-probability characteristics from Eq. (3.10), for systems using coherent signal sources, are shown in Fig. 2. Here the solid lines designate the potential performance of digital systems with various size alphabets, where noise-free counting-type detectors are employed and additive noise is negligible. The dashed curves, taken from another paper (Ref. 10), describe the potential performance of some typical coherent-carrier phase-modulated systems where the additional noise introduced by ideal physical preamplification is taken into account. It is interesting to note that the binary π-differential phase-modulation system has precisely the same theoretical characteristic curve (when it is used with preamplifiers) as does a binary frequency-shift (or pulse-position shift) system using ideal photon-counting detectors without received-signal preamplification.

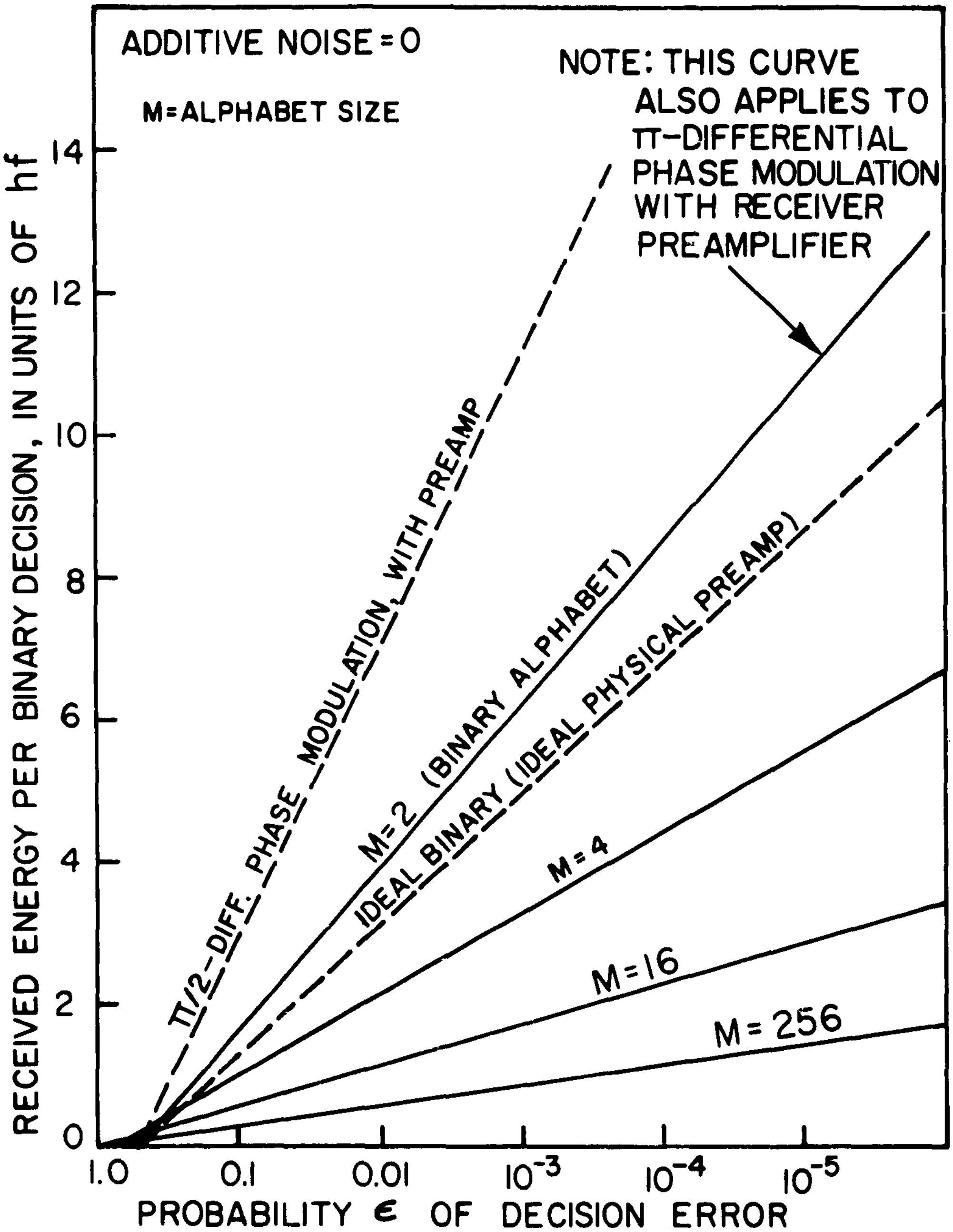

Fig. 2 Received Signal Energy Versus Error Probability (for Coherent Carrier)

The relative consequences of using "incoherent" band-limited thermal noise as a carrier are effectively illustrated, for the noiseless case, by expressing error probabilities of Eqs. (3.10) and (3.5) in slightly different forms. Let N_C and N_I designate the total average numbers of received quanta per pulse for systems using coherent and incoherent carrier-signal sources respectively. Thus, Eq. (3.5) may be rewritten

$$\epsilon_I = \left(\frac{M-1}{M}\right)\left[\frac{1}{\bar{n}_{R_I} + 1}\right]^{\frac{N_I}{\bar{n}_{R_I}}} \qquad (3.11)$$

and Eq. (3.10) may be rewritten as

$$\epsilon_C = \left(\frac{M-1}{M}\right)\left\{\frac{1}{\exp\left[N_C\left(\frac{\bar{n}_{R_I}}{N_I}\right)\right]}\right\}^{\frac{N_I}{\bar{n}_{R_I}}} \qquad (3.12)$$

By equating (3.11) and (3.12), one finds that equal error probabilities for the incoherent and coherent signal sources requires that

$$\frac{N_I}{N_C} = \frac{\bar{n}_{R_I}}{\log_e\left(\bar{n}_{R_I} + 1\right)} \qquad (3.13)$$

Thus, the relative power required by the two alternatives, for any given values of M and ϵ, is a function only of the mean excitation per degree-of-freedom of the received

thermal-noise pulse. The relation in Eq. (3.13), which has been plotted in Fig. 3, reveals that the penalty for incoherence is negligible when the proposed systems are operated with small excitation per dof of the received signal; the penalty becomes sizable as $\bar{n}_R$ grows large. The uncertainty resulting from incoherence in a received pulse can be "averaged out" by dividing available pulse energy among a diversity of degrees-of-freedom in much the same manner as the uncertainty of a fading multipath channel can be reduced by using various forms of diversity transmissions. Under some conditions it might be desirable to segment the receiving antenna aperture and to use separate counters on each segment in order to reduce the effective value of $\bar{n}_R$.

If ideal physical preamplifiers are used prior to detection, additional transmitter power is required to overcome the effects of spontaneous noise emission in the amplifiers. In this case the degradation with respect to a system using an ideal coherent pulse with photon-counting detectors depends slightly on the particular values of M and ϵ involved. Various curves taken from Ref. 9 under the conditions of Eqs. (3.7) and (3.8) fall within the shaded region of Fig. 3. The optimum region of three received photons per d o f [or $3 \times (n_N + 1)$ photons per d o f if additive noise is present] is of considerable significance for systems of this kind.

The relative degradation caused by spontaneous emission of physical preamplifiers is significantly reduced as additive noise becomes appreciable. In Fig. 4 a comparison is made between the performance of systems using ideal photon-counting detectors without preamplifiers and systems using ideal physical preamplification prior to signal detection. This figure applies to a noise-like carrier signal and an average additive noise level of one photon per degree-of-freedom. The solid lines for the photon-counting detector are computed from Eq. (3.3), while the dashed lines for the other receiver are taken from Ref. 9. Note that the effective SNR for the counting detector is $A\bar{n}_T/\bar{n}_N$, equal to six, while that for the amplifying receiver corresponds to

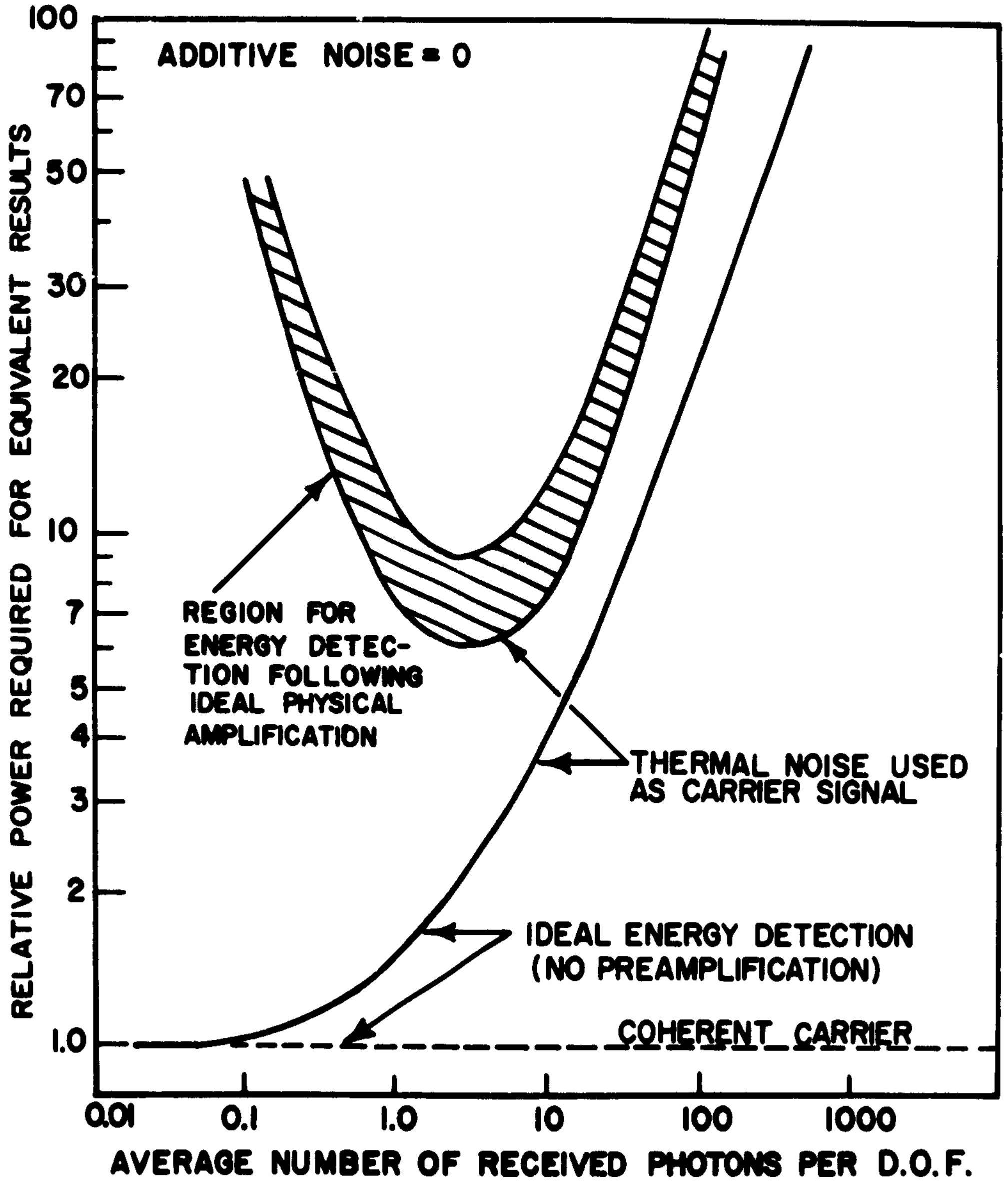

Fig. 3 Effect of Average Received-Signal Excitation on System Performance

$An_T/(n_N + 1)$, which is only three. Whereas the relative power penalty paid for spontaneous emission in optimized systems with zero additive noise is nearly 9 db, as indicated in Fig. 3, under the conditions of Fig. 4 the penalty is only about 1 db.*

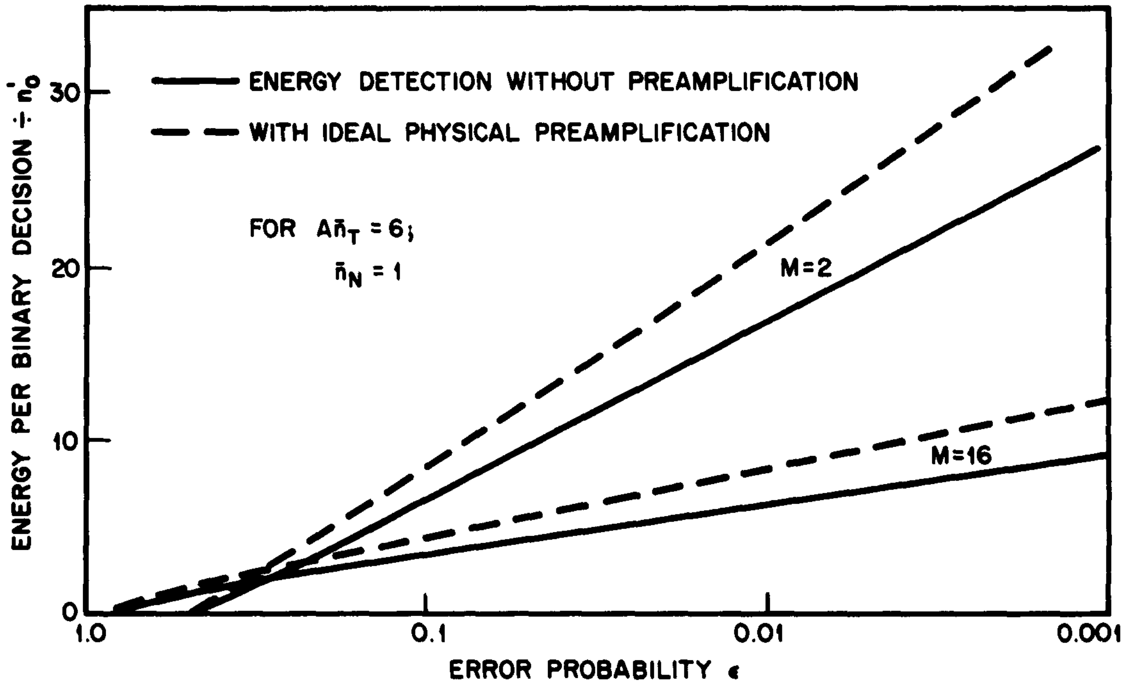

Fig. 4 Relative Performance of Receivers in the Presence of Additive Noise (for Noise-Like Carrier)

3.5 COMPARISON OF RESULTS WITH THEORETICAL PERFORMANCE LIMITS

The information rate that can be communicated over a given channel is generally a function of (a) the mean received signal power, (b) the bandwidth available for occupancy, and (c) the particular modulation and detection techniques that are employed.

*This comparison is slightly misleading because the effective SNR of 3 applicable to Fig. 4 for the preamplification case is optimum, whereas the effective SNR of 6 for the ideal quantum-counting detector case of Fig. 4 is greater than the optimum value.

It is revealing to graph the relation* between (a) and (b) for specified modulation and detection techniques and also for hypothetical situations where modulation and detection may be completely optimized.

Where ideal physical receiving preamplifiers are used on a channel with (or without) additive thermal noise, the minimum theoretical received energy-per-bit of received information, derived from Shannon's equations (Refs. 5, 11) is

$$\text{Received energy/bit} \geqq \text{cycles/bit}\left(2^{\text{bits/cycle}}-1\right) \text{ units of } n_o' \tag{3.14}$$

The relation of Eq. (3.14), expressed in units of $n_o + hf$, is designated "Shannon's Limit" in Fig. 5. Where preamplification is used, this limit is applicable for any level of independent additive noise, including zero. In a case where $n_o \gg hf$, the effects of noise introduced by the amplifier are negligible, and thus Eq. (3.14) is also an accurate limit for systems involving received signal detection without preamplification.

In a situation where additive noise is negligible and preamplification is not used, the effects of quantum disturbances are particularly interesting. A lower limit to the received energy-per-bit (in photons) may be derived as a function of bandwidth occupancy using Gordon's results (Ref. 3). A manipulation of Gordon's expression for the information capacity of a received signal, in the "noiseless" case, yields

$$\text{Energy/bit} \geqq \frac{\bar{n}_R}{\log_2\left[\dfrac{\left(\bar{n}_R+1\right)^{\left(\bar{n}_R+1\right)}}{\left(\bar{n}_R\right)^{\bar{n}_R}}\right]} \text{ photons.} \tag{3.15}$$

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*It is again convenient to normalize with respect to information rates so that energy-per-bit is expressed as a function of cycles-per-bit.

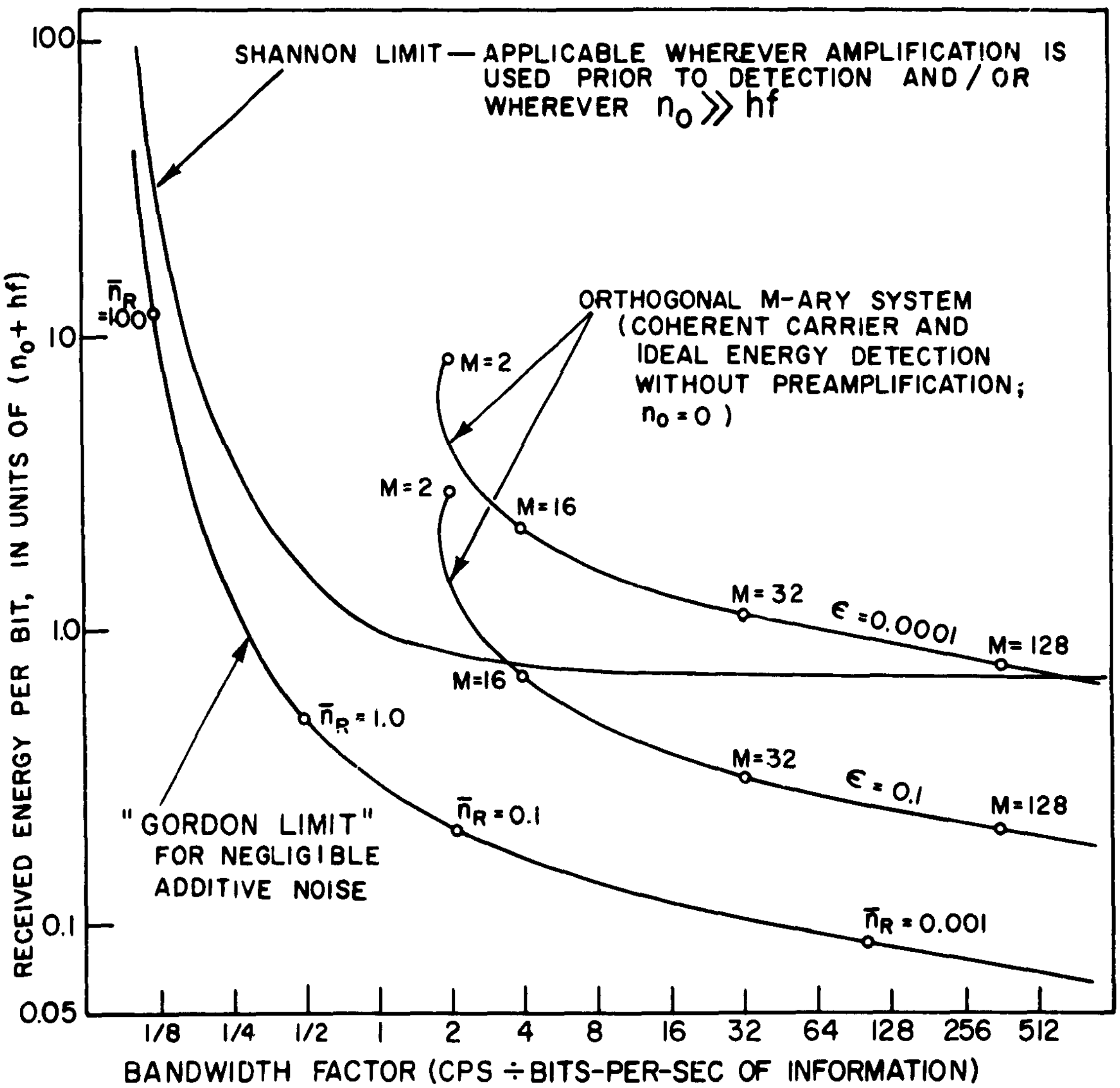

Fig. 5 Required Energy-Per-Bit of Information Versus Bandwidth Factor

and

$$\text{Cycles/bit} \geqq \frac{1}{\log_2\left[\frac{\left(\bar{n}_R+1\right)^{\left(\bar{n}_R+1\right)}}{\left(\bar{n}_R\right)^{\bar{n}_R}}\right]} \tag{3.16}$$

The limiting curve defined by the equalities of Eqs. (3.15) and (3.16) has been designated as "Gordon's Limit" in Fig. 5. An apparent conclusion drawn from this limit is that the information that can be conveyed per unit of received energy is unlimited in spite of quantum disturbances if there is no additive noise. Practically speaking, of course, the necessary alphabet sizes may become unreasonable, available bandwidth (within the "narrowband" approximation that implies constant photon energies) is not unlimited, and the effects of a very small additive thermal noise level become less negligible, other factors remaining constant, when one tries to take undue advantage of the situation. Nevertheless, it is clear that quantum effects do not inherently limit the information that can be communicated per unit of received signal energy in the manner that finite additive noise does.

For detection without prior amplification when additive noise is negligible, the digital modulation techniques considered in this paper exhibit the same sort of unlimited performance characteristics as does Gordon's limit. For a coherent carrier signal source (or an incoherent source with small average received-signal mode excitation) the required received signal energy-per-bit (taken from Eq. (3.10)) may be expressed as

$$\text{Energy/bit} = \frac{\text{Energy per decision}}{\text{Bits per decision}} = \frac{\frac{M}{M-1}\log_e(\epsilon)}{\log_2(M)} \tag{3.17}$$

while the required cycles-per-bit are given by*

$$\text{Cycles/bit} = \frac{M}{\log_2(M)} \tag{3.18}$$

A minor correction (Ref. 10), in most cases, must be applied to Eqs. (3.17) and (3.18) to reflect information loss due to non-zero error probabilities. Curves for different error probabilities and a wide range of alphabet sizes have been included in Fig. 5. Note that the performance of these relatively crude techniques can be much better, in terms of power economy, than the theoretical optimum for systems using ideal physical receiving preamplifiers, no matter how sophisticated the modulation and detection schemes of the latter may become.

*For the noise-like carrier, a greater bandwidth will generally be required.

Section 4
CONCLUSIONS

The communication potentials of some elementary digital communications techniques have been investigated using a quantum-mechanical channel model that is appropriate where discrete phenomena assume significant proportions. These techniques appear to be particularly attractive when the degree of coherence of submillimeter and optical signal sources is limited.

Where additive noise is negligible, it has been found that the performance of systems using elementary pulse-position (and frequency-shift) modulation schemes and photon-counting detectors can be made insensitive to imperfections in signal coherence. Furthermore, the potential performance of these systems (in terms of power required per unit of information-transfer rate)is potentially superior to that offered by any system of modulation which involves received-signal amplification prior to detection.

As additive noise becomes appreciable, the degrading effects of ideal physical pre-amplification in the receivers of systems rapidly diminish. The ultimate performance of communications systems operating at extremely short wavelengths can be bounded by curves based jointly on the work of Shannon and that of Gordon.

Section 5
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